OCS Algebra I ● Unpacked Content

For the new Common Core standards that will be effective in all North Carolina schools in the 2012-13.

What is the purpose of this document?
To increase student achievement by ensuring educators understand what the standards mean a student must know and be able to do completely and comprehensively.

What is in the document?
Descriptions of what each standard means a student will know and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure that description is helpful, specific and comprehensive.

How do I send Feedback?
We intend the explanations and examples in this document to be helpful, specific and comprehensive. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at feedback@dpi.state.nc.us and we will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
You can find the standards alone at www.corestandards.org.
The Real Number System

Common Core Cluster

Extend the properties of exponents to rational exponents

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **square root, radical, exponent, base, cube root**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
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</table>
| **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. | **N-RN.1** Students understand the relationship between square roots and exponents and cube roots and exponents. Example 1: \[
\sqrt{9} \text{ is } 3 \text{ because } 3 \cdot 3 \text{ or } 3^2 = 9 \\
\text{If } 5^2 = 25, \text{ then the } \sqrt[3]{25} \text{ is } 5 \\
\sqrt[3]{8} \text{ is } 2 \text{ because } 2 \cdot 2 \cdot 2 = 2^3 = 8 \\
\text{If } 4^3 = 64 \text{ then the } \sqrt[3]{64} \text{ is } 4
\]|
## Quantities

### Common Core Cluster

**Reason quantitatively and use units to solve problems**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **square _____, cubic _____, axis, scale, origin, y-axis, x-axis, quantity, accuracy**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
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</table>
| N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | N-Q.1 Use units as a tool to help solve multi-step problems. Students should use the units assigned to quantities in a problem to help identify which variable they correspond to in a formula. Based on the type of quantities represented by variables in a formula, choose the appropriate units to express the variables and interpret the meaning of the units in the context of the relationships that the formula describes. Example 1:  
1. Which unit of measure would be appropriate for the area of a circle with a diameter of 2 feet?  
   a. square feet  
   b. feet  
   c. cubic feet  
2. Which use of measure would be appropriate for the radius of a circle?  
   a. square feet  
   b. feet  
   c. cubic feet  
Given a graph or data display, read and interpret the scale and origin. Solution:  
1. Choice a, square feet is appropriate for area  
2. Choice b, feet is appropriate for radius (linear measure)  
Example 2: The graph below represents the price of the bananas at one store.  
1. What does the origin represent?  
2. Identify the scale on the y-axis.  
3. What do the points on the line represent? |
**Solution:**
1. Zero pounds cost zero dollars.
2. The scale on the $y$-axis represents an increase of 20 cents.
3. The price of pounds of bananas. For example, the point at 2 represents 2 pounds sell for 50 cents.

![Graph of Cost of Bananas](image)

<table>
<thead>
<tr>
<th>Pounds</th>
<th>Price (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
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</tbody>
</table>

**N-Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**N-Q.2** Define the appropriate quantities to describe the characteristics of interest for a population. For example, if you want to describe how dangerous the roads are, you may choose to report the number of accidents per year on a particular stretch of interstate. Generally speaking, it may not be appropriate to report the number of exits on that stretch of interstate to describe the level of danger.

**Examples:**
1. What quantities could you use to describe the best city in North Carolina?
2. What quantities could you use to describe how good a basketball player is?

**Solutions:**
1. Possible quantities include population, area, price of houses, etc.
2. Possible quantities include free throw percentage, number of assists, points scored per game, etc.
<table>
<thead>
<tr>
<th>N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</th>
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</table>
| **N-Q.3** Understand that the tool used determines the level of accuracy that can be reported for a measurement. For example, when using a ruler, you can only legitimately report accuracy to the nearest division. If a ruler is used that has centimeter divisions to measure the length of my pencil, I can only report its length to the nearest centimeter.

**Example 1:**
What is the accuracy of a ruler with 16 divisions per inch?

**Solution:**
\[
\frac{1}{16} \text{ of an inch}
\]
| Common Core Standard | Unpacking | A-SSE.1 Interpret expressions that represent a quantity in terms of its context. 

a. Interpret parts of an expression, such as terms, factors, and coefficients.  

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \( P(1+r)^n \) as the product of \( P \) and a factor not depending on \( P \). |
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<thead>
<tr>
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<tbody>
<tr>
<td>A-SSE.1a. Students manipulate the terms, factors, and coefficients in expressions to explain the meaning of the individual parts of the expression.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Example 1: 
Joan is participating in the annual fundraising race. Her mother agreed to give her $5 and then $0.50 for each mile she runs. The expression \( 0.50m + 5 \) represents the amount her mother would pay. 
1. What information does the 5 give about the problem?  
2. What information does the 0.50 give about the problem?  
3. What does the \( m \) represent? |
| Solution: 
1. The 5 represents the amount raised if 0 miles are ran.  
2. The 0.50 represents the amount earned for each mile ran.  
3. The \( m \) represents the number of miles ran. |
| Example 2: 
A virus cell divides into two cells every 4 hours. The expression \( 5(2^x) \) represents the number of cells that exist after any number of divisions if there were 5 cells in the beginning. 
1. What information does the 5 give about the problem?  
2. What information does the 2 give about the problem?  
3. What does the \( x \) represent? |
| Solution: 
1. The 5 represents the number of cells before any divisions.  
2. The 2 means that each division produces two new cells.  
3. The \( x \) represents the number of times the divisions occur. |

Note: At this level, limit to linear expressions, exponential expressions with integer exponents and quadratic expressions.
Example 3:
The length of a rectangle is 3 more than its width. Let \( w \) represent the width. The expression, \( w(w + 3) \), represents the area of the rectangle.

1. What does the term \( w + 3 \) represent?

Solution:
The term \( w + 3 \) represents the length of the rectangle.

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see \( x^4 - y^4 \) as \( (x^2)^2 - (y^2)^2 \), thus recognizing it as a difference of squares that can be factored as \( (x^2 - y^2)(x^2 + y^2) \).

A-SSE.2 Students rewrite algebraic expressions by combining like terms or factoring to reveal equivalent forms of the same expression.

Example 1:
Write an equivalent expression for \( 3(x + 5) - 2 \).

Solution:
\[
3x + 15 - 2 = 3x + 13
\]
Distribute the 3
Combine like terms

Example 2:
Write equivalent expressions for: \( 3a + 12 \).

Solution:
Possible solutions might include factoring as in \( 3(a + 4) \), or other expressions such as \( a + 2a + 7 + 5 \).

Example 3:
Factor \( x^2 - 2x - 15 \)

Solution:
\( (x - 5)(x + 3) \)
### Seeing Structure in Expressions

#### Common Core Cluster

**Write expressions in equivalent forms to solve problems**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **equivalent, zeroes of a quadratic function,**

<table>
<thead>
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<tr>
<td>A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*</td>
<td>A-SSE.3a Students factor quadratic expressions and find the zeros of the quadratic function they represent. Zeroes are the x-values that yield a y-value of 0. Students explain the meaning of the zeros as they relate to the problem.</td>
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<tr>
<td><strong>Note:</strong> At this level, the limit is quadratic expressions of the form $ax^2 + bx + c$</td>
<td><strong>Example 1:</strong> The length of a rectangle is 3 m greater than its width. The area of the rectangle is 54 m². Find the length and width.</td>
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<tr>
<td><strong>Solution:</strong> Let $w = \text{width}$ $w + 3 = \text{length}$</td>
<td>Area is length time width so the equation would be: $w(w + 3) = 54$ $w^2 + 3w = 54$ Distribute the $w$ $w^2 + 3w - 54 = 54 - 54$ Subtract 54 from both sides $w^2 + 3w - 54 = 0$ Factor the expression $(w - 9)(w + 6) = 0$ $w - 9 = 0$ or $w + 6 = 0$ Set each factor equal to 0 $w = 9$ or $w = -6$ Solve for $w$</td>
</tr>
<tr>
<td>The width must be a positive number so the width of this rectangle would be 9. The length would be three more or 12.</td>
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**OCS Algebra I**

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# Arithmetic with Polynomials and Rational Expressions

## Common Core Cluster

**Perform arithmetic operations on polynomials**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **like terms, polynomial, coefficient, constant**

## Common Core Standard

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>A-APR.1</td>
<td>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. <strong>Note:</strong> At this level, limit to addition and subtraction of quadratics and multiplication of linear expressions.</td>
</tr>
<tr>
<td>A-APR.1</td>
<td>Add, subtract, and multiply polynomials. <strong>At this level, limit to addition and subtraction of quadratics and multiplication of linear expressions.</strong></td>
</tr>
</tbody>
</table>

### Example 1:

Write an equivalent expression for \((x^2 + 3x + 4) - (x^2 + x)\)

**Solution:**

\[ (x^2 + 3x + 4) - x^2 - x \]

\[ x^2 + 2x + 4 \]

### Example 2:

Find the perimeter of the triangle to the right:

**Solution:**

\[ x + 3 + x + 3 + x = 3x + 6 \]

### Example 3:

Multiply \((x + 3)(x + 2)\)

**Solution:**

\[ (x + 3)(x + 2) = x^2 + 5x + 6 \]
### Creating Equations

#### Common Core Cluster

Create equations that describe numbers or relationships

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rule, equation, inequality**

<table>
<thead>
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| A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | A-CED.1 Students write an equation or inequality in one variable to model a contextual situation. Students define the variable and use appropriate units.  
(Note: The focus for this standard is writing the equations and inequalities; A-REI.1 and A-REI.3 focus on solution methods) |

**Note:** At this level, focus on linear and exponential functions.

#### Writing Linear Equations

You are ordering tulip bulbs from a catalog. One bulb is $0.75. Shipping is $3.00. How many bulbs can be bought for $30.

**Solution:**

Let \( x \) = the number of tulip bulbs

\[
3 + 0.75x = 30
\]

\[
x = 36
\]

#### Writing Linear Inequalities

The admission price to the fair is $10. Ride tickets cost $3 each. You only have $40 in cash to spend on admission and rides. Write an inequality and solve it to find how many ride tickets you can buy.

**Solution:**

Let \( x \) = the number of ride tickets

\[
10 + 3x \leq 40
\]

\[
x \leq 10
\]
Exponential Example:
(equations only)

Example 5:
A cell divides into two cells every hour. Write an equation to find how much time it would take to have 1024 cells.

Solution:
Let \(x\) = number of hours
\[2^x = 1024\]

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Note: At this level, focus on linear, exponential and quadratic. Limit to situations that involve evaluating exponential functions for integer inputs.

A-CED.2 Given a contextual situation, write equations in two variables that represent the relationship that exists between the quantities. Graph the equations.

Writing linear equations
The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost \(c\) of renting a calculator as a function of the number of months, \(m\).

Solution:
\[c = 10 + 5m\]

Writing exponential equations
A population of 50 insects triples in size every month. Write a rule to model the population after \(x\) months.

Solution:
\[y = 50(3^x)\]

Writing quadratic equations
A wading pool is being built with an area of 45 ft\(^2\). The length is 4 more than the width. What are the dimensions of the wading pool.

Solution:
\[w(w + 4) = 45\]
### A-CED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**Note:** At this level, limit to linear equations and inequalities

### A-CED.4

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.*

**Note:** At this level, limit to formulas that are linear in the variable of interest, or to formulas involving squared or cubed variables.

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### A-CED.3

Write a system of equations and/or inequalities to solve a real world problem. Describe the solutions in context.

#### System of Linear Equations

**Example 1:**
The sum of two numbers is 15. The difference of the same two numbers is 5. What are the two numbers?

**Solution:**

\[
\begin{align*}
  x + y &= 15 \\
  x - y &= 5 
\end{align*}
\]

#### System of Linear Inequalities

**Example:**
Susan has a job in an ice cream shop and a babysitting job.
- The job in the ice cream shop pays $6 an hour.
- The babysitting job pays $4 an hour
- Susan wants to earn at least $60 per week
- Susan cannot work more than 12 hours per week.

Write a system of inequalities to represent this situation. Graph it to show all possible solutions.

**Solution:**

Let $x =$ hours worked in the ice cream shop
Let $y =$ hours worked babysitting

\[
\begin{align*}
  x + y &\leq 12 \\
  6x + 4y &\geq 60 
\end{align*}
\]

---

### A-CED.4

Solve multi-variable formulas or literal equations, for a specific variable. Explicitly connect this to the process of solving equations using inverse operations.

**Example 1:**
Solve for $l$ in the following formula: $A = l \cdot w$

**Solution:**

\[
A = l \cdot w
\]

Divide both sides by $w$ to get $l$ by itself

\[
\frac{A}{w} = l
\]
### Reasoning with Equations and Inequalities

<table>
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<tr>
<th>Common Core Cluster</th>
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<tbody>
<tr>
<td>Understand solving equations as a process of reasoning and explain the reasoning</td>
<td>A-REI.1 Students solve equations by understanding that the two sides of equation represent the same amount (are balanced). To maintain the balance, the same math operations must be done to both sides of an equation. Students are able to explain how they solve equations.</td>
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<tr>
<td>A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
<td>Example 1: Solve: $3x - 4 = 14$ Solution: $3x - 4 = 14$ $3x - 4 = 14$ $+4 +4$ Add to both sides $\frac{3}{3} x = \frac{18}{3}$ Divide both sides by 3 $x = 6$</td>
</tr>
<tr>
<td>Common Core Cluster</td>
<td>Unpacking</td>
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<tr>
<td><strong>Solve equations and inequalities in one variable</strong></td>
<td><strong>What does this standard mean that a student will know and be able to do?</strong></td>
</tr>
<tr>
<td><strong>A-REI.3</strong> Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
<td><strong>A-REI.3</strong> Students solved one-step and two-step equations in the Introduction to High School Mathematics class. These equations could be reviewed prior to solving equations with the variable on both side of the equations. <strong>Solving Equations:</strong> Students solve one-variable equations including those with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. <strong>Example 1:</strong> Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved to $x = 4$. This means that when the value of $x$ is 4, both sides will be equal. <strong>Connection to systems unit:</strong> If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be $(4, 17)$.</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>$10 \cdot 4 - 23 = 29 - 3 \cdot 4$</td>
</tr>
<tr>
<td></td>
<td>$40 - 23 = 29 - 12$</td>
</tr>
<tr>
<td></td>
<td>$17 = 17$</td>
</tr>
<tr>
<td><strong>Example 2:</strong> Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for $x$ that will make the sides equal. <strong>Combine like terms</strong> <strong>Add 7x to each side</strong></td>
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<tr>
<td></td>
<td>$x + 7 + 6x = 19 + 7x$</td>
</tr>
<tr>
<td></td>
<td>$7x + 7 = 19 + 7x$</td>
</tr>
<tr>
<td></td>
<td>$7 \neq 19$</td>
</tr>
<tr>
<td>This solution means that no matter what value is substituted for $x$ the final result will never be equal to each other.</td>
<td></td>
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</tbody>
</table>
Example 3: An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of $x$ will produce a valid equation. For example the following equation, when simplified will give the same values on both sides.

$4 (6a + 1) = 4 + 24a$

$24a + 4 = 4 + 24a$

[Connection to systems unit: If each side of the equation were treated as a linear equation and graphed, the graph would be the same line.]

**Solving Inequalities:**
Students solved one-step and two-step inequalities in the Introduction to High School Mathematics class. (no negative coefficients)

Example 4:

$6x - 15 < 4x + 11$
## Reasoning with Equations and Inequalities

### Common Core Cluster

**Solve systems of equations**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **substitution, system of equations, system of inequalities, intersection, parallel lines**

<table>
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<tbody>
<tr>
<td>A-REI.5</td>
<td>Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</td>
</tr>
</tbody>
</table>

### Unpacking

What does this standard mean that a student will know and be able to do?

**A-REI.5** Solve a system of equations using elimination. Limit to systems where the coefficients of one variable are opposites (Example 1) or the coefficients of one variable are the same and can be eliminated by multiplying one of the equations by -1 (Example 2). Write final answers as ordered pairs.

**Example 1:**

Solve the following system of equations:

\[
\begin{align*}
2x + y &= 9 \\
3x - y &= 11
\end{align*}
\]

**Solution:**

\[
\begin{align*}
2x + y &= 9 \\
3x - y &= 11
\end{align*}
\]

\[
\begin{align*}
2x + y &= 9 \\
3x - y &= 11
\end{align*}
\]

\[
\begin{align*}
5x &= 20 \\
5x &= 20
\end{align*}
\]

\[
\begin{align*}
x &= 4
\end{align*}
\]

The ordered pair (4, 1) will solve both equations.

**Example 2:**

Solve the following system of equations:

\[
\begin{align*}
3x + y &= 6 \\
x + y &= 2
\end{align*}
\]
Solution:

\[ 3x + y = 6 \]
\[ (x + y = 2) \times (-1) \]

\[ 3x + y = 6 \]
\[ -x - y = -2 \]
\[ 2x = 4 \]
\[ x = 2 \]

Multiply both sides of the 2\textsuperscript{nd} equation by -1

\[ 3(2) + y = 6 \]
\[ 6 + y = 6 \]
\[ y = 0 \]

Subtract 6 from both sides

Substitute 2 for \( x \)

The ordered pair (2, 0) will solve both equations.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Example 1: (one solution)

Graph the following system and identify the point of intersection.

\[ y = 3x - 4 \]
\[ y = -3x + 2 \]

Solution:

The solution of this system is (1, -1). Replacing the variables in the equations with these values will make true statements.

\[ y = 3x - 4 \]
\[ y = -3x + 2 \]
\[ -1 = 3(1) - 4 \]
\[ -1 = -3(1) + 2 \]
\[ -1 = 3 - 4 \]
\[ -1 = -3 + 2 \]
\[ -1 = -1 \]
\[ -1 = -1 \]
Example 2: (no solution)

\[ y = \frac{5}{4}x - 2 \]
\[ y = \frac{5}{4}x - 1 \]

Solution:

The lines are parallel, which means that no one ordered pair will solve both equations.
### Reasoning with Equations and Inequalities

#### Common Core Cluster

**Represent and solve equations and inequalities graphically**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **solution, boundary line**

<table>
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<tbody>
<tr>
<td>A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
<td><strong>A-REI.10</strong> Students understand that all points on the graph of a two-variable equation are solutions to the equation. Substituting the values into the equation will make a true statement. <strong>Note:</strong> At this level, focus on linear and exponential equations.</td>
</tr>
</tbody>
</table>

**Example 1 (Linear Equation):**
The equation $y = 2x + 3$ is graphed. Which of the following points are on the graph? How do you know?
- a. (2, 7)
- b. (1, 6)
- c. (-3, -3)

**Solution:**
Points “a” and “c” are solutions to the equation and would be on the line representing this equation. Substituting the values back into the equation makes a true statement. Point “b” does not make a true statement.

\[
\begin{align*}
y &= 2x + 3 \\
7 &= 2(2) + 3 \\
7 &= 4 + 3 \\
7 &= 7 \checkmark
\end{align*}
\]

\[
\begin{align*}
y &= 2x + 3 \\
6 &= 2(1) + 3 \\
6 &= 2 + 3 \\
6 &\neq 5
\end{align*}
\]

\[
\begin{align*}
y &= 2x + 3 \\
-3 &= 2(-3) + 3 \\
-3 &= -6 + 3 \\
-3 &= -3 \checkmark
\end{align*}
\]

**Example 2 (Exponential Equation):**
The equation $y = 2^x$ is graphed. Which of the following points are on the graph? How do you know?
- a. (0, 1)
- b. (3, 8)
- c. (1, 0)
Solution:
Points “a” and “b” are solutions to the equation and would be on the line representing this equation. Substituting the values back into the equation makes a true statement. Point “c” does not make a true statement.

\[ y = 2^x \]
\[ 1 = 2^0 \]
\[ 8 = 2^3 \]
\[ 0 = 2^1 \]

\[ y = 2^x \]
\[ 8 = 2^3 \]
\[ 0 = 2^1 \]

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Note: At this level, focus on linear and exponential functions.

A-REI.11 Students understand that solving a one-variable equation of the form \( f(x) = g(x) \) is the same as solving the two-variable system \( y = f(x) \) and \( y = g(x) \).

Example 1:
The system, \( y = 3x - 4 \) \[ \text{[Think of this equation as } y = f(x) \] \[ y = -3x + 2 \] \[ \text{[Think of this equation as } y = g(x) \] could be solved by setting the functions equal to each other creating the equation \( 3x - 4 = -3x + 2 \) \[ f(x) = g(x) \].
Solving this equation gives \( x = 1 \). When this value is substituted into each equation, the value of \( y \) will be -1.

A-REI.11 Solve systems by making tables for each side of the equation. The x-value that makes the two sides equal is the solution to the equation. Students explain the meaning of the solution of the system.

Example 2:
The math club is trying to decide where to get T-shirts printed.
- Sam’s shirt shop charges $20 for setup and then $5 for each T-shirt printed.
- T-Shirts Unlimited charges $10 for each T-shirt

Which company should the seniors choose?

Solution:

<table>
<thead>
<tr>
<th>Number of T-shirts</th>
<th>Cost (Sam’s Shirts)</th>
<th>Cost (T-Shirts Unlimited)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>50</td>
</tr>
</tbody>
</table>

For less than 4 shirts T-Shirts Unlimited is the best buy. Sam’s Shirts is the best price for 5 or more shirts. For 4 shirts the cost is the same. If graphed (4, 40) would be the intersection of the two lines, indicating that 4 shirts cost
A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A-REI.12 Students graph an inequality and identify when the boundary line should be dashed (< or >) or solid (≤ or ≥). Students recognize that all the points on the half-plane (shaded part) are solutions to the linear inequality.

**Linear Inequalities in Two Variables**

**Example 1:**
Graph \( y > x + 4 \).
Is the boundary line dashed or solid? How do you know?
Prove that \((1, 7)\) is part of the solution set.

**Solution:**
The line is dashed since the inequality is “greater than” instead of “greater than or equal to”.

\((1, 7)\) is part of the solution set because if the values are substituted into the inequality a true statement is made:

A-REI.12 Students understand that the solutions to a system of inequalities in two-variables are the points that lie in the intersection of the corresponding half-planes.

**System of Inequalities**

**Example 2:**
Graph the following system of inequalities.
\[
\begin{align*}
y &\leq \frac{1}{2}x + 2 \\
y &> 3x - 3
\end{align*}
\]
Solution:

[Diagram of a geometric figure]
<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-IF.1</strong> Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If ( f ) is a function and ( x ) is an element of its domain, then ( f(x) ) denotes the output of ( f ) corresponding to the input ( x ). The graph of ( f ) is the graph of the equation ( y = f(x) ).</td>
<td><strong>F-IF.1</strong> A function occurs when each input ( (x) ) has only one output ( (y) ). Given a table, equation, or graph, students determine if the relation is a function. Students understand the domain is the set of ( x ) values and the range is the set of ( y ) values. In a function, the notation ( f(x) ) is used for ( y ). <strong>Graphs</strong> Students recognize graphs such as the one below is a function using the vertical line test, showing that each ( x )-value has only one ( y )-value; whereas, graphs such as the following are not functions since there are 2 ( y )-values for multiple ( x )-value.</td>
</tr>
</tbody>
</table>
Tables or Ordered Pairs
Students read tables or look at a set of ordered pairs to determine functions and identify equations where there is only one output (y-value) for each input (x-value).

Functions
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
</tr>
</tbody>
</table>

Not A Function
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>-4</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>-5</td>
</tr>
</tbody>
</table>

{(0, 2), (1, 3), (2, 5), (3, 6)}

Equations
Students recognize equations such as $y = x$ or $y = x^2 + 3x + 4$ as functions; whereas, equations such as $x^2 + y^2 = 25$ are not functions.

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Note: At this level, the focus is linear and exponential functions.

F-IF.2 Students recognize $f(x)$ function notation. Students evaluation functions for different inputs.

Example 1: (linear)
Evaluate $f(x) = 2x + 5$ for $x = 4$

Solution:
$f(4) = 2 \cdot 4 + 5$
$f(4) = 8 + 5$
$f(4) = 13$

Example 2: (exponential)
Evaluate $f(x) = 2(3^x)$ for $x = 2$

Solution:
$f(2) = 2(3^2)$
$f(2) = 2(9)$
$f(2) = 18$
Students interpret the meaning of the input and output given a function in context.

Example 3:
A band wants to record and copy a CD. One company charges $250 for recording the CD. There is also a cost of $3 to copy each CD. The total cost $T(c)$ is a function of the number of CDs copied. The function rule $T(c) = 250 + 3c$ represents the cost. Evaluate the function when $c = 10$. What does the answer tell about the situation?

Solution:
$T(c) = 250 + 3c$
$T(10) = 250 + 3(10)$
$T(10) = 250 + 30$
$T(10) = 280$

This means that it will cost $280 to record and copy 10 CDs.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

F-IF.3 A sequence can be thought of as a function, with the input numbers consisting of the integers, and the output numbers being the terms of the sequence. [Connect to arithmetic and geometric sequences (F-BF.2). Emphasize that arithmetic sequences are examples of linear functions; geometric sequences are examples of exponential functions.

Arithmetic Sequences (linear functions)
In an arithmetic sequence, each term is obtained from the previous term by adding the same number each time. This number is called the common difference. For example, in the sequence 3, 7, 11, 15, 19, … the common difference is 4 since 4 is added to get the next term. As the sequence continues to grow, each term will have a unique value making it a function.

Geometric Sequences (exponential functions)
In a geometric sequence, each term is obtained from the previous term by multiplying by a constant amount, called the common ratio. For example, in the sequence 3, 12, 48, … each term is multiplied by 4, the common ratio. As the sequence continues to grow, each term will have a unique value making it a function.
### Interpreting Functions

**Common Core Cluster**

Interpret functions that arise in applications in terms of the context

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: function, domain, average rate of change, \( y \)-intercept, \( x \)-intercept, increasing, decreasing, positive, negative, maximum, minimum, symmetry

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-IF.4</strong> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <strong>Key features include:</strong> intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
<td></td>
</tr>
<tr>
<td><strong>F-IF.4</strong> Given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the graph in the context of the problem. For a linear function, the characteristics described should include rate of change, ( x )-intercept, and ( y )-intercept.</td>
<td></td>
</tr>
<tr>
<td><strong>Note:</strong> At this level, focus on linear, exponential and quadratic functions; no end behavior or periodicity.</td>
<td><strong>F-IF.4</strong> Given either a linear, quadratic or exponential function, students identify key features in graphs and/or tables.</td>
</tr>
</tbody>
</table>

### Linear Functions

**F-IF.4** Given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the graph in the context of the problem. For a linear function, the characteristics described should include rate of change, \( x \)-intercept, and \( y \)-intercept.

**Example 1:**
The math club collected $120. They are spending $20 each month for their meeting supplies and refreshments. The table below represents the amount remaining at the end of each month.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

a. What is the \( y \)-intercept and what does it mean in the context of the problem?
b. What is the \( x \)-intercept and what does it mean in the context of the problem?
c. What is the rate of change and what does it mean in the context of the problem?
Solution:

a. The \( y \)-intercept is \((0, 120)\). This is the starting amount.
b. The \( x \)-intercept is \((6, 0)\). After 6 weeks the account will have $0.
c. The rate of change is -20. Each month the amount decreases $20.

**Exponential Functions**

**Example 2:**

a. What is the \( y \)-intercept of the graph below?
b. What is the common ratio?

d. [Graph of an exponential function]

**Solution:**

a. The \( y \)-intercept is \((0, 3)\). This represents the starting amount.
b. The common ratio is 2. When \( x \) is 1, the \( y \)-value is 6; when \( x \) is 2, the \( y \)-value is 12. The change in the \( y \)-values as the \( x \) increases by 1 indicates a common ratio of 2.

**Quadratic Functions**

From a graph or table of a quadratic function, students identify the roots (\( x \)-intercepts), line of symmetry, the vertex and if the graph has a maximum value or minimum value.

**Example 3:**

Identify the following on the graph to the right:

a. Vertex
b. Root(s)
c. Line (axis) of symmetry
d. Maximum or minimum

d. [Graph of a quadratic function]
Solution:
  a. Vertex: (0, -25)
  b. Roots: (-5, 0) and (4, 0)
  c. Line of symmetry: x = 0
  d. Minimum graph

Students use calculators to create tables and answer question in context.

Example 4:
A rocket is launched from 180 feet above the ground at time \( t = 0 \). The function that models this situation is given by \( h(t) = -16t^2 + 96t + 180 \), where \( t \) is measured in seconds and \( h \) is height above the ground measured in feet.
  a. What is the height of the rocket two seconds after it was launched?
  b. What is the maximum value of the function and what does it mean in context?
  c. When is the rocket 260 feet above the ground?

Solution:
  a. 308 feet
  b. The maximum value of the function is (3, 324). This means that after 3 seconds the rocket was at a maximum height of 324 feet.
  c. At 2 seconds and 5 seconds

F-IF.4 Given a verbal description of the relationship between two quantities, students sketch a graph of the relationship, showing key features.

Example 4:
Elizabeth and Joshua tried to get a monthly allowance from their mother. Their mother initially paid them one penny, then gave them 2 pennies for the first day of the month, 4 pennies for the second day, 8 pennies for the third day and so on. How much would their mother have to pay on the 10th day of the month?
Sketch the graph of the relationship between the two quantities and explain what the point (0, 1) represents.

Solution:
The point (0, 1) represents the starting amount.
| **F-IF.5** | **F-IF.5** Students identify an appropriate domain from a graph based on the context. Students also identify the meaning of a point in terms of the context.  
**Example 1:** Jennifer’s cell phone plan charges her $20 each month for the phone and $0.10 for each minute she is on the phone. What would be the appropriate domain that describes this relationship? Describe the meaning of the point (10, 21).  
**Solution:** The set of positive integers would be an appropriate domain since there cannot be a negative number of minutes and parts of minutes are not charged. The point (10, 21) means that the charge for 10 minutes of service would be $21.  
**Example 2:** What is the domain in the graph to the right?  
**Solution:** A context is not given for the graph so the domain would be the set of Real numbers. |
|---|---|
| **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.  
**Note:** At this level, focus on linear functions and exponential functions whose domain is a subset of the integers. | **F-IF.6** Estimate the average rate of change over a specified interval of a function from the function’s graph.  
**Example 1 (Linear Function):** The graph below shows the distance a car travels over 3 hours. What is the average rate of change between $x = 1$ and $x = 2$? |
Solution:
At $x = 1$ the distance is 50 miles. At $x = 2$ the distance is 100 miles. Fifty (50) miles were traveled in one hour so the average rate of change is 50 miles for every 1 mile.

Example 2 (Exponential Function):
Each year a local tennis tournament starts with 64 participants. During each round, half of the players are eliminated. Create a table to show the number of participants remaining after each round. What is the average rate of change between the 2nd and 3rd round?

Solution:

<table>
<thead>
<tr>
<th>Round</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of players remaining</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

The average rate of change between rounds 2 and 3 is $\frac{1}{2}$. 
## Interpreting Functions

### Common Core Cluster

**F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-IF.7a</td>
<td>Students graph linear and quadratic functions expressed symbolically and show key features of the graph. Graph simple cases by hand, and use technology to show more complex problem. For linear functions, students identify the slope and y-intercept. For quadratic functions, students identify the vertex, whether the graph is a maximum or minimum graph, and the x-intercepts or roots.</td>
</tr>
</tbody>
</table>

**Example 1:**

Use the graph at the right to answer the following questions:

- a. What is the y-intercept?
- b. What is the rate of change?

**Solution:**

- a. The y-intercept is (0, 4)
- b. The rate of change is $-\frac{1}{2}$

**Example 2:**

The function $x^2 - 8x + 15$ is graphed to the right.

- a. What is the vertex?
- b. What are the x-intercepts?
- c. Is this a maximum or minimum graph?

**Solution:**

- a. (4, 1)
- b. (3, 0) and (5, 0)
- c. This is a minimum graph.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**Note:** At this level, for part e, focus on exponential functions only.

**F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**Note:** At this level, only factoring expressions of the form \( ax^2 + bx + c \), is expected. Completing the square is not addressed at this level.

**F-IF.7e** Students graph exponential functions identifying the \( y \)-intercept and rate of change.

**Example 1:**

a. What is the \( y \)-intercept of the graph to the right?

b. Does this graph represent exponential growth or exponential decay?

c. What is the rate of change?

**Solution:**

a. The \( y \)-intercept is (5, 0)

b. The graph represents exponential decay?

c. The rate of change is \( \frac{1}{2} \).

**F-IF.8** Students factor a quadratic function to identify zeros and interpret them in the context of the problem.

**Example 1:**

The function \( x^2 - 4x - 12 \) represents a flowerbed with an area of 12 and side lengths \( x \) feet and \( x - 4 \) feet. What are the lengths of the sides?

**Solution:**

Factor \( x^2 - 4x - 12 \) to get \( (x - 6)(x + 2) \). Setting each of these terms equal to 0 will give values of \( x \) as either 6 or -2. Since length cannot be negative, \( x \) must equal 6. The lengths of the sides are 6 feet and 6 – 2 or 4 feet.
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)^{\frac{t}{10}}, \) and classify them as representing exponential growth or decay.

F-IF.8b Students identify exponential functions as growth or decay.

Example 1:
Classify each exponential function below as either a growth function or decay function.

a. \( y = 4^t \)

b. \( y = 0.25^t \)

c. \( y = 10 \left( \frac{1}{2}^t \right) \)

d. \( y = \frac{1}{2} \left( \frac{1}{6}^t \right) \)

Solution:

a. Growth  
b. Decay  
c. Decay  
d. Growth

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Note: At this level, focus on linear, exponential, and quadratic functions.

F-IF.9 Students compare the properties of two functions represented by verbal descriptions, tables, graphs, and equations.

Linear Functions
Example 1:
Compare the following functions to determine which has the greater rate of change.
Function 1: \( y = 2x + 4 \)

Function 2:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Solution: The rate of change for function 1 is 2; the rate of change for function 2 is 3. Function 2 has the greater rate of change.
Exponential Functions
Example 2:
Compare the following functions to determine which shows the greatest growth rate?

Function 1:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>81</td>
</tr>
</tbody>
</table>

Quadratic Functions
Example 3:
Compare the functions represented below. Which has the greatest minimum value?

Function 1:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-7</td>
</tr>
<tr>
<td>-1</td>
<td>-12</td>
</tr>
<tr>
<td>0</td>
<td>-15</td>
</tr>
<tr>
<td>1</td>
<td>-16</td>
</tr>
<tr>
<td>2</td>
<td>-15</td>
</tr>
<tr>
<td>3</td>
<td>-12</td>
</tr>
</tbody>
</table>
Building Functions

Common Core Cluster

Build a function that models a relationship between two quantities

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: arithmetic sequence, geometric sequence, constant, coefficient, rule

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-BF.1 Write a function that describes a relationship between two quantities.</td>
<td></td>
</tr>
<tr>
<td>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</td>
<td></td>
</tr>
</tbody>
</table>

F-BF.1a Students write a function to describe relationships between two quantities for linear or exponential functions.

Linear Function

Example 1:
The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write a rule for the total cost of renting a calculator as a function of the number of months (m).

Solution:

\[ f(m) = 10 + 5m \quad \text{or} \quad c = 10 + 5m \]

Students could also write a recursive function \( NEXT = NOW + 5 \), starting at 10 or show the calculations for determining the cost for any number months:

<table>
<thead>
<tr>
<th>Months</th>
<th>Calculations</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>10 + 5</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>10 + 5 + 5</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>10 + 5 + 5 + 5</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>10 + 5 + 5 + 5 + 5</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>10 + 5 + 5 + 5 + 5 + 5</td>
<td>35</td>
</tr>
<tr>
<td>( M )</td>
<td>10 + 5m*</td>
<td>( C ) or ( f(m) )</td>
</tr>
</tbody>
</table>

*Note that the 10 remains constant in all the calculations. The number of fives varies, corresponding to the number of months the calculator is being rented.
Exponential Function

Example 2:
A certain species of bacteria in a laboratory begins with 25 cells and doubles every 10 minutes. Write a rule for the number of bacteria as a function of the number of 10-minute periods (x).

Solution:
\( f(x) = 25 \cdot 2^x \) or \( y = 25 \cdot 2^x \)

Students could also write a recursive function \( \text{NEXT} = \text{NOW} \cdot 2 \), starting at 25 or show the calculations for determining the cost for any number months:

<table>
<thead>
<tr>
<th>Number of 10-minute periods</th>
<th>Calculations</th>
<th>Number of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>25 \cdot 2</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>25 \cdot 2 \cdot 2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>25 \cdot 2 \cdot 2 \cdot 2</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>25 \cdot 2 \cdot 2 \cdot 2 \cdot 2</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>25 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2</td>
<td>800</td>
</tr>
<tr>
<td>( x )</td>
<td>25 \cdot 2^x</td>
<td>( y ) or ( f(x) )</td>
</tr>
</tbody>
</table>

*Note that the 25 remains constant in all the calculations. The number of twos varies, corresponding to the number of 10-minute periods.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Note: At this level, limit to addition or subtraction of constant to linear.

F-BF.1b Students create a function to model a relationship between two quantities. Students then determine the value of \( x \) that will give a specified \( y \) value.

Example 1:
A pot of water with a temperature of 70° is placed on a burner. The temperature of the water increases 2° every minute. The function \( y = 70 + 2x \) represents the relationship between the number of minutes and the temperature. When the temperature reaches 100° (\( y = 100 \)), the pot must be removed from the burner. In how many minutes should the pot be removed?

Solution:
When \( x = 15 \) the value of \( y \) will be 100. The pot must be removed after 15 minutes.
### exponential or quadratic functions or addition of linear functions to linear or quadratic functions.

**Example 2:**
To create tickets, Sue takes 1 sheet of paper and tears it into two equal pieces (1 tear = 2 tickets). The 2 tickets are then stacked and torn again so that 4 tickets are created after 2 tears. The function \( y = 2^x \) represents the number of tickets created after \( x \) tears. How many tears are needed to produce 16 tickets (where would the function \( y = 16 \) intersect with the exponential function?)

**Solution:**
When \( x = 4 \) (after 4 tears), the number of tickets will be 16.

**Example 3:**
A rocket is launched from 180 feet above the ground at time \( t = 0 \). The function that models this situation is given by the quadratic function \( h(t) = -16t^2 + 96t + 180 \), where \( t \) is measured in seconds and \( h \) is height above the ground measured in feet. At what times would the constant function \( y = 260 \), intersect the graph.

**Solution:**
When \( t = 2 \) seconds and 5 seconds the height is 260 feet.

### F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situation, and translate between the two forms.

**Note:** At this level, formal recursive notation is not used. Instead, use of informal recursive notation (such as \( \text{NEXT} = \text{NOW} + 5 \) starting at 3) is intended.

**F-BF.2** Students write the recursive and explicit forms of the arithmetic and geometric sequences, translate between the recursive and explicit forms and use the recursive and explicit forms of arithmetic and geometric sequences to model real-world situations.

#### Arithmetic Sequences (linear functions)
In an arithmetic sequence, each term is obtained from the previous term by adding the same number each time. This number is called the common difference. For example, in the sequence 3, 7, 11, 15, 19… the common difference is 4 since 4 is added to get the next term.

NOW-NEXT equations show to calculate the value of the next term in a sequence from the value of the current term. The arithmetic sequence \( \text{NEXT} = \text{NOW} \pm C \) is the recursive form of a linear function. The common difference \( C \) corresponds to the slope \( m \) in the slope-intercept form of a linear function, \( y = mx + b \). The initial value of the sequence corresponds to the y-intercept, \( b \).

**Example 1:**
The car rental company charges a one-time $25 fee for the car’s navigation system (GPS), then $45 a day.

- a. How much would it cost to rent the car for 3 days?
- c. Write a recursive (NOW-NEXT) rule to find the cost of the rental for \( x \) days.
- d. Convert this rule into a linear equation.
b. Write an equation to calculate the cost for a 30-day rental.

Solution:
- a. $25 + $45 + $45 + $45 = $160
- b. NEXT = NOW + 45, starting at 25
- c. \( y = 25 + 45x \)
- d. \( y = 25 + 45(30) \)

Geometric Sequences (exponential functions)

In a geometric sequence, each term is obtained from the previous term by multiplying by a constant amount, called the common ratio. For example, in the sequence 3, 6, 12, 24, 48… the common ratio is 2 since each term is 2 times the previous term. Also, the ratios \( \frac{6}{3}, \frac{12}{6}, \frac{24}{12} \) are equal to the common ratio of 2.

NOW-NEXT equations show how to calculate the value of the next term in a sequence from the value of the current term. The geometric sequence \( \text{NEXT} = B \times \text{NOW} \) is the recursive form of an exponential function. The common ratio \( B \) corresponds to the base \( b \) in the explicit form of an exponential function, \( y = ab^x \). The initial value corresponds to the \( y \)-intercept, \( a \).

Example 2:

A single bacterium is placed in a test tube and divides into two bacteria after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues for one hour until test tube is filled up.
- a. How many bacteria are produced in the test tube at the 5-minute mark?
- b. Write a recursive (NOW-NEXT) rule to find the number of bacteria in the test tube after \( x \) minutes.
- c. Convert this rule into an exponential equation.
- d. Write an equation to find how many bacteria are produced in the test tube at the one-hour mark?

Solution:
- a. There are \( 1 \times 2 \times 2 \times 2 \times 2 = 32 \) bacteria
- b. NEXT = NOW \( \times 2 \), starting at 1
- c. \( y = 2^x \)
- d. \( y = 2^{60} \)
## Building Functions

### Common Core Cluster

**Build new functions from existing functions**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: *vertical, horizontal, positive, negative, translate, origin*

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-BF.3</strong> Identify the effect on the graph of replacing ( f(x) ) by ( f(x) + k, k f(x), f(kx), ) and ( f(x + k) ) for specific values of ( k ) (both positive and negative); find the value of ( k ) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</td>
<td></td>
</tr>
<tr>
<td><strong>F-BF.3</strong> Know that when adding a constant, ( k ), to a function, it moves the graph of the function vertically. If ( k ) is positive, it translates the graph up, and if ( k ) is negative, it translates the graph down.</td>
<td></td>
</tr>
</tbody>
</table>

**Linear Functions:**

**Example 1:**

Susan makes a graph of \( y = x \).

- If the function changes to \( y = x + 3 \), how does the graph change?
- How does the graph change if the function changes from \( y = x \) to \( y = x - 6 \)?

**Solution:**

- The line would cross the \( y \)-axis at 3 instead of the origin.
- The line would cross the \( y \)-axis at -6 instead of the origin.

**Note:** At this level, limit to vertical and horizontal translations of linear and exponential functions. Even and odd functions are not addressed.

**Exponential Functions:**

**Example 2:**

A graph of the exponential function \( y = 2^x \) is made. How is the function \( y = 2^x + 3 \) different?

**Solution:** The graph has moved up the \( y \)-axis 4 spaces since 3 is added to \( 2^0 \) which is equal to 1.

**Example 3:**

Horizontal movements with exponential functions can be explored by using a graphing calculator. Values can be added to or subtracted from the exponent of \( x \) to observe the effect. Values that are added \( (y = 2^{x+3}) \) will move the graph of the function to the left; values that are subtracted \( (y = 2^{x-3}) \) will move the graph of the function to the right.
Linear, Quadratic, and Exponential Models

**Common Core Cluster**

**Construct and compare linear, quadratic, and exponential models and solve problems**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: linear function, exponential function.

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions</td>
<td>F-LE.1 Students classify situations as linear or exponential based on the change between intervals. Students recognize that linear functions change by equal differences and that exponential functions change by equal factors.</td>
</tr>
<tr>
<td>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</td>
<td>Example 1: For each situation below, Decide whether it can be represented using a linear model or an exponential model.</td>
</tr>
<tr>
<td>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
<td>a.</td>
</tr>
<tr>
<td>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td>
<td>b.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

d. One person does good deeds for three new people. Then the three new people each do good deeds for three more new people. Next, nine people each do a good deed for three more new people, and so on.

e. NEXT = NOW + 4, starting at 20

f. 4, 8, 16, 32, 64, …

g. Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential.
<table>
<thead>
<tr>
<th>F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</th>
</tr>
</thead>
</table>

**F-LE.2** Students identify the rate of change and initial value ($y$-intercept) from arithmetic or geometric sequences, NOW-NEXT statements, tables, graphs, or verbal descriptions to write a linear or exponential function. Students understand that the function represents the relationship between the $x$-value and the $y$-value; what math operations are performed with the $x$-value to give the $y$-value.

### Linear Functions

#### Tables:

Students recognize that in a table the $y$-intercept is the $y$-value when $x$ is equal to 0. The slope can be determined by finding the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ between the change in two $y$-values and the change between the two corresponding $x$-values.

#### Example 1:

Write an equation that models the linear relationship in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Solution:** The $y$-intercept in the table below would be $(0, 2)$. The distance between 8 and -1 is 9 in a negative direction $\rightarrow$ -9; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or $\frac{y}{x}$

or $\frac{-9}{3} = -3$. The equation would be $y = -3x + 2$

#### Graphs:

Using graphs, students identify the $y$-intercept as the point where the line crosses the $y$-axis and the slope as the rise, run.
Example 2:
Write an equation that models the linear relationship in the graph below.

Solution: The y-intercept is 4. The slope is \( \frac{1}{4} \), found by moving up 1 and right 4 going from (0, 4) to (4, 5). The linear equation would be \( y = \frac{1}{4} x + 4 \).

Contextual Situations:
In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem.

Example 3:
The company charges $45 a day for the car as well as charging a one-time $25 fee for the car’s navigation system (GPS). Write a function for the cost in dollars, \( c \), as a function of the number of days, \( d \), the car was rented.

Solution: \( C = 45d + 25 \)

Students interpret the rate of change and the y-intercept in the context of the problem. In Example 3, the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help students model contextual situations.

Exponential Functions
Example 4:
Suppose a single bacterium lands in a cut on your hand. It begins spreading an infection by growing and splitting into two bacteria every 10 minutes. The table below represents the number of bacteria in the cut after several 10-minute intervals.

<table>
<thead>
<tr>
<th>Number of 10-minute periods</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria Count</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>128</td>
</tr>
</tbody>
</table>
a. Use NOW-NEXT to write a rule relating the number of bacteria at one time to the number 10 minutes later.

b. Write an equation showing how the number of bacteria can be calculated from the number of stages in the growth and division process.

Solution:

a. NEXT = NOW \cdot 2, starting at 1.

b. \( y = 2^x \)

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Note: At this level, limit to linear, exponential, and quadratic functions; general polynomial functions are not addressed.

F-LE.3 Using graphs and tables, students understand that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Example 1:

Jim was offered three allowance plans.

Plan 1: Receive 1 cent on the first day and add 2 cents each day.

Plan 2: Receive 1 cent on the first day and double the amount each day.

Plan 3: Use his dad’s formula \( x^2 + 2x + 1 \), where \( x \) is the day of the month.

Which plan should Jim use to earn the most money? Explain your answer.

Solution:

<table>
<thead>
<tr>
<th>Plan 1</th>
<th>Plan 2</th>
<th>Plan 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>Amount</td>
<td>Day</td>
</tr>
<tr>
<td></td>
<td>Cents</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>19</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>23</td>
<td>12</td>
</tr>
</tbody>
</table>
### Common Core Cluster

**Interpret expressions for functions in terms of the situation they model**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **domain, practical values, non-practical values**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
<th><strong>F-LE.5</strong> Interpret the parameters in a linear or exponential function in terms of a context.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>F-LE.5</strong> Understand the difference between the practical and the non-practical values of the domain in linear and exponential situations and explain their meaning in terms of their context.</td>
<td></td>
</tr>
<tr>
<td>Example 1:</td>
<td><strong>Solution:</strong> Based on this scenario, the values could not be negative. The variable, ( n ), represents the number of years and years could not have negative values.</td>
<td></td>
</tr>
</tbody>
</table>

---

The function \( f(n) = P(1.08)^n \) is used to model the amount of money in a savings account that earns 8\% interest, compounded annually. \( N \) is the number of years since the initial deposit, \( P \). Could the values of \( n \) (the domain) ever be negative? Why or why not?
<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-CO.1</td>
<td>Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</td>
</tr>
</tbody>
</table>
| Note: At this level, distance around a circular arc is not addressed. | G-CO.1 Know that a point has position, no thickness or distance. A line is made of infinitely many points, and a line segment is a subset of the points on a line with endpoints. A ray is defined as having a point on one end and a continuing line on the other.  
An angle is determined by the intersection of two rays.  
A circle is the set of infinitely many points that are the same distance from the center forming a circular arc, measuring 360 degrees.  
Perpendicular lines are lines that intersect at a point to form right angles.  
Parallel lines that lie in the same plane and are lines in which every point is equidistant from the corresponding point on the other line. The lines will never intersect. |
### Expressing Geometric Properties with Equations

**G-GPE**

**Common Core Cluster**

**Use coordinates to prove simple geometric theorems algebraically**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: congruent, parallel, perpendicular, slope, perimeter, area, midpoint, coordinates, x-coordinate, y-coordinate, mean, Pythagorean Theorem, reciprocals

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point ((1, \sqrt{3})) lies on the circle centered at the origin and containing the point ((0, 2)).</td>
<td>G-GPE.4 Students use the concepts of slope (parallel and perpendicular) and distance (length of line segments) to prove that a figure in the coordinate system is a special geometric shape. Students identify a missing coordinate to create a specified geometric shape.</td>
</tr>
</tbody>
</table>

**Example 1:**

A quadrilateral is a parallelogram if a pair of its opposite sides are congruent and parallel. A figure has the following coordinates:

\[
\begin{align*}
A (0, 0) & \quad B (2, 5) & \quad C (8, 3) & \quad D (6, -2)
\end{align*}
\]

- a. Find the length of side \(AB\).
- b. Find the length of side \(CD\).
- c. Find the slope of side \(AB\).
- d. Find the slope of side \(CD\).

**Solution:**

- a. The length of side \(AB\) is \(\sqrt{29}\) or 5.39
- b. The length of side \(CD\) is \(\sqrt{29}\) or 5.39
- c. The slope of side \(AB\) is \(\frac{5}{2}\)
- d. The slope of side \(CD\) is \(\frac{5}{2}\)

Because \(AB\) and \(CD\) are congruent and have the same slope (parallel, the figure is a parallelogram.)
G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.5 Use the formula for the slope of a line to determine whether two lines are parallel or perpendicular. Two lines are parallel if they have the same slope and two lines are perpendicular if their slopes are opposite reciprocals of each other. In other words the product of the slopes of lines that are perpendicular is (-1). Find the equations of lines that are parallel or perpendicular given certain criteria.

**Example 1:**

1. Use the line graphed to the right to answer the following questions:
   a. What would be the slope of a line parallel to this line?
   b. What would be the slope of a line perpendicular to this line?

   **Solution:**
   a. \( \frac{2}{3} \)
   b. \( -\frac{3}{2} \)

**Example 2:**

What is the equation of the line parallel to the line \( y = 2x + 4 \) and passing through the point (0, -3)?

**Solution:**
The slope of the line parallel would be 2. The point (0, -3) is the y-intercept. The equation would be \( y = 2x - 3 \).

**G-GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**G-GPE.6** Given two points on a line, find the point that divides the segment into two equal parts. If finding the mid-point, it is always halfway between the two endpoints. The x-coordinate of the mid-point will be the mean of the x-coordinates of the endpoints; the y-coordinate will be the mean of the y-coordinates of the endpoints.

**Example 1:**

Jennifer and Jane are best friends. On a coordinate grid of the town, Jennifer’s house is at (9, 7) and Jane’s house is at (15, 9). If they want to meet in the middle, what are the coordinates of the place they should meet?

---

Unpacked Content
Solution:
Find the mean of the x-coordinates: \( \frac{9 + 15}{2} \) to get 12.

Find the mean of the y-coordinates: \( \frac{7 + 9}{2} \) to get 8.

The meeting should occur at the coordinate (12, 8).

**G-GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

**G-GPE.7** Students find the perimeter of polygons and the area of triangles and rectangles using coordinates on the coordinate plane.

**Example 1:**
John was visiting three cities that lie on a coordinate grid at (-3,0), (3, 0), and (0, 4). If he visited all the cities and ended up where he started, what is the distance in miles he traveled (1 unit = 1 mile)

**Solution:**
The distance from (-3, 0) to (3, 0) is 6 miles.
The distance from (3, 0) to (0, 4) is 5 miles:

*Draw a triangle connecting these points and use the Pythagorean Theorem to find the hypotenuse.*
The distance from (0, 4) to (-3, 0) is 5 miles:

*Draw a triangle connecting these points and use the Pythagorean Theorem to find the hypotenuse.*
The perimeter would be 6 miles + 5 miles + 5 miles or 16 miles.

**Example 2:**
A triangle has the following coordinates:

A (-1, 2)    B (-1, 6)    C (2, 2)

What is the area of the triangle?

**Solution:**
This is a graph of a right triangle. Students can use the coordinates (-1, 2) and (-1, 6) to determine a height of 4 units. The length of the base is from (-1, 2) to (2, 2) which is 3 units. To find the area multiply the base times the height and divide by 2 to get 6 units².
# Geometric Measurement and Dimension

## Common Core Cluster

### Explain volume formulas and use them to solve problems

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **circle, circumference, area, volume, cylinder, pyramid, cone, spheres**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.</td>
<td><strong>G-GMD.1</strong> Students understand how formulas for circumference of a circle, area of a circle and volume of a cylinder, pyramid and cone relate to the figure.</td>
</tr>
</tbody>
</table>

**Note:** Informal limit arguments are not the intent at this level.

### Circumference of a Circle

Students understand the formula for the circumference of a circle by measuring the diameter and then taking this length and going around the outside of the circle (circumference). The diameter will go around a little over 3 times, which indicates the . The radius will go around half of the circle a little over 3 times. This amount is represented by Pi, \( \pi \). This relationship gives the formula \( C = \pi d \). Since the diameter is twice the radius (2\( r \)), the formula for circumference can also be expressed as \( C = 2\pi r \).

### Area of a Circle

Understanding the formula for the area of a circle can be shown using dissection arguments. First dissect portions of the circle like pieces of a pie. Arrange the pieces into a curvy parallelogram as indicated below.

\[
A_{\text{rect}} = \text{Base} \times \text{Height} \\
A_{\text{area}} = \frac{1}{2} (2\pi r) \times r \\
A_{\text{area}} = \pi r \times r \\
A_{\text{area}} = \pi r^2
\]

Volume of a Cylinder
Students build on understandings of circles and volume from previous grades/courses to find the volume of cylinders, finding the area of the base $\pi r^2$ and multiplying by the number of layers (the height).

\[
\frac{9 + 15}{2} \quad V = \pi r^2 h
\]

find the area of the base and multiply by the number of layers

Volume of a Cone
Students understand that the volume of a cylinder is 3 times the volume of a cone having the same base area and height or that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder having the same base area and height.

\[
V = \frac{\pi r^2 h}{3}
\]

Volume of a Pyramid
Students understand that the volume of a prism is 3 times the volume of a pyramid having the same base area and height or that the volume of a pyramid is $\frac{1}{3}$ the volume of a prism having the same base area and height.

To find the volume of a pyramid, find the area of the base, multiply by the height and then divide by three.

\[
V = \frac{Bh}{3} \quad B = \text{Area of the Base} \quad h = \text{height of the pyramid}
\]
**G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

**G-GMD.3** Solve problems using volume formulas for cylinders, pyramids, cones, and spheres. Formulas will be given.

**Example 1:**
James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter’s volume.

![Diagram of a cylindrical planter](image)

**Solution:**

\[
V = \pi r^2 h
\]

\[
V = 3.14 (40)^2 (100)
\]

\[
V = 502,400 \text{ cm}^3
\]

The answer could also be given in terms of \( \pi \):

\[
V = 160,000 \pi
\]

**Example 2:**
A candle mold is in the shape of a square pyramid. The length of the sides is 3 inches and the height is 5 inches. How much wax is needed for each candle?

**Solution:**

\[
V = \frac{bh}{3}
\]

The area of the base is 3 in. x 3 in. or 9 in\(^2\)
Example 3:
How many cubic centimeters of water would be needed to fill the cone to the right?

Solution:

\[ V = \frac{1}{3} \pi r^2 h \]

\[ V = \frac{1}{3} \pi (3^2)(5) \]

\[ V = \frac{1}{3} \pi (45) \]

\[ V = 15 \pi \text{ cm}^3 \]

Example 4:
Approximately, how much air would be needed to fill a soccer ball with a radius of 14 cm?

Solution:

\[ V = \frac{4}{3} \pi r^3 \]

\[ V = \frac{4}{3} (3.14)(14^3) \]

\[ V = 11.5 \text{ cm}^3 \]
# Interpreting Categorical and Quantitative Data

**Common Core Cluster**

Summarize, represent, and interpret data on a single count or measurement variable

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **dot plot, histogram, box plot, median, mean, outlier, interval, interquartile range, lower quartile, upper quartile, standard deviation, cluster, gaps, outliers**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).</td>
<td><strong>S-ID.1</strong> Students display data graphically using number lines. Dot plots, histograms and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others.</td>
</tr>
<tr>
<td></td>
<td>Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.</td>
</tr>
<tr>
<td></td>
<td>A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a histogram. Note that changing the size of the interval changes the appearance of the graph and the conclusions may vary from it.</td>
</tr>
<tr>
<td></td>
<td>A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data.</td>
</tr>
</tbody>
</table>

*OCS Algebra I*

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Unpacked Content
Example 1:
Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Solution:
[6-Trait Writing Rubric Diagram]

- Most students scored a 3.
- The minimum score was 0; the maximum score was 6.
- The data is pulled to the left since only a few students scored a 0 or 1.

Example 2:
Students were collecting data to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

<table>
<thead>
<tr>
<th>11</th>
<th>21</th>
<th>5</th>
<th>12</th>
<th>10</th>
<th>31</th>
<th>19</th>
<th>13</th>
<th>23</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11</td>
<td>25</td>
<td>14</td>
<td>34</td>
<td>15</td>
<td>14</td>
<td>29</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>26</td>
<td>23</td>
<td>12</td>
<td>27</td>
<td>4</td>
<td>25</td>
<td>15</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>12</td>
<td>39</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>28</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
A histogram using 5 intervals 0-9, 10-19, …30-39) to organize the data is displayed below.

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.
Example 3:  
Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

<table>
<thead>
<tr>
<th>Age in Months</th>
<th>Age in Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>130</td>
</tr>
<tr>
<td>131</td>
<td>131</td>
</tr>
<tr>
<td>132</td>
<td>132</td>
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<tr>
<td>133</td>
<td>133</td>
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<tr>
<td>134</td>
<td>134</td>
</tr>
<tr>
<td>136</td>
<td>136</td>
</tr>
<tr>
<td>137</td>
<td>137</td>
</tr>
<tr>
<td>138</td>
<td>138</td>
</tr>
<tr>
<td>139</td>
<td>139</td>
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<td>142</td>
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<td>142</td>
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<td>143</td>
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<td>144</td>
<td>144</td>
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<tr>
<td>145</td>
<td>145</td>
</tr>
<tr>
<td>147</td>
<td>147</td>
</tr>
<tr>
<td>149</td>
<td>149</td>
</tr>
<tr>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

Solution:  
**Five number summary**  
Minimum – 130 months  
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months  
Median (Q2) – 139 months  
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months  
Maximum – 150 months

This box plot shows that  
- ¼ of the students in the class are from 130 to 132.5 months old  
- ¼ of the students in the class are from 142.5 months to 150 months old  
- ½ of the class are from 132.5 to 142.5 months old  
- The median class age is 139 months.
S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.2 Students compare two sets of data using measures of center (mean and median) and variability (Standard Deviation and IQR). Students understand which measure of center and which measure of spread is most appropriate to describe a given data set. The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.

Example 1:
The two data sets below depict random samples of the management salaries in two companies. Based on the salaries below which measure of center will provide the most accurate estimation of the salaries for each company?
- Company A: 1.2 million, 242,000, 265,500, 140,000, 281,000, 265,000, 211,000
- Company B: 5 million, 154,000, 250,000, 250,000, 200,000, 160,000, 190,000

Solution:
The median would be the most accurate measure since both companies have one value in the million that is far from the other values and would affect the mean.

S-ID.2 Select the appropriate measures to describe and compare the center and spread of two or more data sets in context.

Example 2:
Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 10 days, she measured the heights (in cm) of each set of seedlings. The data she collected is shown below. Which fertilizer is best? Use graphs and/or statistics to justify your answer.

<table>
<thead>
<tr>
<th>Fertilizer A</th>
<th>Fertilizer B</th>
<th>Fertilizer C</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>11.0</td>
<td>10.5</td>
</tr>
<tr>
<td>6.3</td>
<td>9.2</td>
<td>11.8</td>
</tr>
<tr>
<td>1.0</td>
<td>5.6</td>
<td>15.5</td>
</tr>
<tr>
<td>5.0</td>
<td>8.4</td>
<td>14.7</td>
</tr>
<tr>
<td>4.5</td>
<td>7.2</td>
<td>11.0</td>
</tr>
<tr>
<td>5.2</td>
<td>12.1</td>
<td>10.8</td>
</tr>
<tr>
<td>3.2</td>
<td>10.5</td>
<td>13.9</td>
</tr>
<tr>
<td>4.6</td>
<td>14.0</td>
<td>12.7</td>
</tr>
<tr>
<td>2.4</td>
<td>15.3</td>
<td>9.9</td>
</tr>
<tr>
<td>5.5</td>
<td>6.3</td>
<td>10.3</td>
</tr>
<tr>
<td>3.8</td>
<td>8.7</td>
<td>10.1</td>
</tr>
<tr>
<td>1.5</td>
<td>11.3</td>
<td>15.8</td>
</tr>
<tr>
<td>6.2</td>
<td>17.0</td>
<td>9.5</td>
</tr>
<tr>
<td>6.9</td>
<td>13.5</td>
<td>13.2</td>
</tr>
<tr>
<td>2.6</td>
<td>14.2</td>
<td>9.7</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th></th>
<th>Fertilizer A</th>
<th>Fertilizer B</th>
<th>Fertilizer C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.4</td>
<td>10.9</td>
<td>11.9</td>
</tr>
<tr>
<td>Median</td>
<td>4.6</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

Fertilizer C would be the best. Although B and C have the same median, the mean is higher for C.
S-ID.2 Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.

Example 3:
The box plots below show a comparison of heights for males and females in the 20 – 29 age group. How do the box plots demonstrate the differences in the median heights and the interquartile range for each group?

Solution:
The median height for males is 71 inches; the median height for females is 65 inches. The IQR for males is 4 inches; the IQR for females is 5 inches. The males’ heights center is at a higher value but are slightly more variable.
S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S-ID.3 Students understand and are able to use the context of the data to explain why its distribution takes on a particular shape, possibly due to outliers.

Example 1:

a. Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right?

b. Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left?

c. Why does the shape of the distribution of heights of the students at your school tend to be symmetrical?

Solution:

a. The salaries of a few professional athletes are much higher than most so the graph is skewed right.

b. Most students would have high scores; however, a few scores may be low pulling the graph to the left.

c. There will be some students who are tall and short; however, the majority would be approximately the same height.

S-ID.3 Students understand that the higher the value of a measure of variability, the more spread out the data set is.

Example 2:

On last week’s math test, Mrs. Smith’s class had an average of 83 points with a standard deviation of 8 points. Mr. Tucker’s class had an average of 78 points with a standard deviation of 4 points. Which class was more consistent with their test scores? How do you know?

Solution:

Mr. Tucker’s class has the smaller standard deviation so his scores are most consistent even though the average score is lower.
**S-ID.3** Students understand and can explain the effect of any outliers on the shape of a graph, measures of center, and spread of the data sets.

**Example 3:**
Which of the following graphs would contain outliers? How do you know?

**Graph A:**

**Solution:**
Graph A has outliers since the graph is not symmetrical and skewed to the left.

**Example 4:**
The heights of Washington High School’s basketball players are: 69 in, 64 in, 67 in, 66 in, 65 in, 63 in, and 67 in. A student transfers to Washington High and joins the basketball team. Her height is 82 in.
   a. What affect does her height have on the team’s height center and spread?
   b. How many players are taller than the new mean team height?
   c. Which measure of center most accurately describes the team’s average height? Explain.

**Solution:**
   a. The mean increased from 66 inches to 68 inches; the median increased from 66 inches to 67 inches. The variability, or spread, in the data also increased.
   b. The new team height is 68 inches. There are only two players as tall or taller than this measure.
   c. The median most accurately describes the team’s height since it is not as affected by the height of 82 inches.
## Interpreting Categorical and Quantitative Data

### Common Core Cluster

**Summarize, represent, and interpret data on two categorical and quantitative variables**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **two-way tables, scatter plot, constant, coefficient, residual, linear regression**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S-ID.5</strong> Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</td>
<td><strong>S-ID.5</strong> Students understand that a two-way table provides a way to organize data between two categorical variables. Data for both categories needs to be collected from each subject. Students calculate the relative frequencies to describe associations. Example 1: Twenty-five students were surveyed and asked if they received an allowance and if they did chores. The table below summarizes their responses.</td>
</tr>
<tr>
<td></td>
<td>Of the students who do chores, what percent do not receive an allowance? <strong>Solution:</strong> Of the 20 students who do chores, 75% do not receive an allowance.</td>
</tr>
<tr>
<td><strong>S-ID.6</strong> Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</td>
<td><strong>S-ID.6</strong> Bivariate data refers to two-variable data, one to be graphed on the x-axis and the other on the y-axis. Students represent numerical data on a scatter plot, to examine relationships between variables. They analyze scatter plots to determine if the relationship is linear (positive, negative association or no association) or non-linear. Students should determine if a linear or exponential model best fits a set of data. Fit this type of function to the data and interpret constants and coefficients in the context of the data</td>
</tr>
</tbody>
</table>
Example 1:
A population of single-celled organisms was grown over a period of 16 hours. The number of organisms at a given time is recorded in the table below:

<table>
<thead>
<tr>
<th>Time, hours (x)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Organisms (y)</td>
<td>25</td>
<td>36</td>
<td>52</td>
<td>68</td>
<td>85</td>
<td>104</td>
<td>142</td>
<td>260</td>
</tr>
</tbody>
</table>

a. Using technology, make a scatterplot of the data \((time, number)\).
b. What type of function does this data model?
c. Write a function to model this data.

**Solution:**
- a. See graph
- b. This data models an exponential function.
- c. \(y = 27(1.15)^x\)

b. Informally assess the fit of a function by plotting and analyzing residuals.

**Note:** At this level, for part b, focus on linear models.

S-ID.6b Students use technology to calculate the residuals (Diagnostic On) and recognize that the closer the value is to 1, the stronger the relationship between the two variables. The sign of the residual indicates a positive or negative association.

See examples in S-ID.8

c. Fit a linear function for a scatter plot that suggests a linear association.

**S-ID.6c** Students determine a line of best fit for data set that has a linear association, using technology.

See example in S-ID7
### Interpreting Categorical and Quantitative Data

#### Common Core Cluster

#### Interpret linear models

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **slope, linear model, intercept, correlation coefficient, independent variable, dependent variable**

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S-ID.7</strong> Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td>
<td>S-ID.7 Linear models can be represented with a linear equation. Students interpret the slope and y-intercept of the line in the context of the problem.</td>
</tr>
</tbody>
</table>

**Example 1:**
1. Given data from students’ math scores and absences, make a scatterplot.

![Scatterplot](image1)

2. Draw a linear model paying attention to the closeness of the data points on either side of the line.

![Linear Model](image2)
3. From the linear model, determine an approximate linear equation that models the given data 
   (about \( y = \frac{-25}{3} x + 95 \))

4. Students should recognize that 95 represents the \( y \)-intercept and \( \frac{-25}{3} \) represents the slope of the line. In the context of the problem, the \( y \)-intercept represents the math score a student with 0 absences could expect. The slope indicates that the math scores decreased 25 points for every 3 absences.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.8 Students understand that the correlation coefficient, \( r \), is a measure of the strength and direction of a linear relationship between two quantities in a set of data. The magnitude (absolute value) of \( r \) indicates how closely the data points fit a linear pattern. If \( r = 1 \), the points all fall on a line. The closer \( r \) is to 1, the stronger the correlation. The closer \( r \) is to zero, the weaker the correlation. The sign of \( r \) indicates the direction of the relationship – positive or negative.

Example 1:
The table below gives the fee at a dog grooming business based on the number of services sold. Using technology determine if the relationship is positive or negative and strong or weak based on the correlation coefficient.

<table>
<thead>
<tr>
<th># of services sold</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fee in dollars</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
</tr>
</tbody>
</table>

Solution:
This is a strong, positive correlation as indicated by a \( r \) value of 1.

Example 2:
The athletic department experimented with several different prices on can drinks. The data is shown below:

<table>
<thead>
<tr>
<th>Price per can</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily sales (in cans)</td>
<td>163</td>
<td>130</td>
<td>110</td>
<td>110</td>
<td>75</td>
<td>73</td>
<td>45</td>
</tr>
</tbody>
</table>

a. Using technology, make a scatterplot (price, sales) of the data.
b. Determine the line of best fit.
c. Using technology, identify the correlation coefficient and interpret it’s meaning in the context of the data.
**Solution:**

a. See graph
b. \( y = -180x + 208 \)
c. The correlation coefficient is -0.97, indicating a strong negative correlation. As the price of the cans increases, the number of cans sold decreases.

### S.ID 9 Distinguish between correlation and causation.

**S-ID.9** Students understand that because two quantities have a strong correlation, we cannot assume that the independent variable *causes* a change in the dependent variable. The best method for establishing causation is to conduct an experiment that carefully controls for the effects of other variables.

**Example 1:**
There is a strong positive association between the number of firefighters at a fire and the amount of damage. John said “This means that firefighters must be the *cause* of the damage at a fire.” Is John correct in his reasoning? Explain why or why not.

**Solution:**
John’s reasoning is incorrect since many other factors would affect the amount of damage from a fire.