Assessment of Early Number Sense: An Overview

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Turn to the person next to you…

• Give a clearly operationalized definition of “number sense”
• Discuss as example of how a young child can demonstrate number sense
Is there an innate number sense?

Three and four-year-olds often show mistakes on conservation of number tasks.
Which has more?
What if we used candy and limited reliance on language?

Mehler & Bever, 1967
What if a teddy bear rearranges the row?

McGarrigle & Donaldson, 1974
Are there more roses or flowers?
Newborns

- Infants who are repeatedly shown pictures of two objects become habituated.
- Following habituation, infants look longer at pictures of three objects.

Strauss & Curtis, 1981
Subitizing

• **Perceptual**
  – Apprehension of numerosity without using other mathematical processes (e.g., counting)

• **Conceptual**
  – Apprehension of numerosity through part-whole relationships (*one* three and *one* three form a six on a domino)

Clements, 1999
Abstraction?

- What about the abstract nature of “3”
  - Infants distinguish between two- and three-syllable words
  - Infants suck on a pacifier more vigorously once habituated to a two-syllable word and subsequently provided a three-syllable word
- Controlled for novel words

Bijeljac-Babic, Bertoncini, & Mehler, 1991
If there is a number sense…

What are the implications for assessment and instruction?
Number Sense

• “A child’s fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons.”

Gersten and Chard, 1997
Implications

• Students who enter and leave kindergarten below the 10th percentile
  – 70% remain below the 10th percentile in 5th grade

Morgan et al., 2009
Morgan, Farkas, Maczuga, 2010
Implications

- A meta-analysis of longitudinal data sets indicated **early numeracy** as the strongest predictor of later achievement (above early reading, socio-economic status, and social-emotional functioning)

Duncan et al., 2007
## Implications

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Disabled</th>
<th>Non-Disabled</th>
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<tbody>
<tr>
<td>4th</td>
<td>43%</td>
<td>17%</td>
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<tr>
<td>8th</td>
<td>68%</td>
<td>27%</td>
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<td>12th</td>
<td>83%</td>
<td>36%</td>
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NAEP 2005
Long-term Outcomes

- Innumeracy lowers an adult’s employability & wages, after controlling for poor reading, IQ, & many other relevant factors

Rivera-Batiz, 1992
Corollary to Reading

• To successfully break the cycle:
  – Early identification of need
  – Diagnostic assessment
  – Appropriately matched instruction
  – Frequent monitoring of progress
  – Process monitored for fidelity
Assessment of Number Sense

• **Screening:**
  – Assess foundational core mathematics instruction
  – Early identification of at-risk students

• **Diagnostic Assessment:**
  – Pinpoint areas of strength and need
  – Instructional grouping
  – Development of instructional plan

• **Progress Monitoring:**
  – Evaluate efficacy of instructional plan
  – Make quick decisions regarding adjustment of instructional plan
Screening / Monitoring Number Sense

- Brief
- Efficient
- Inexpensive
- Non-evaluative
- Reliable
- Valid
Screening / Monitoring Number Sense

- Oral Counting
- Number Identification
- Quantity Discrimination
- Strategic Counting
Oral Counting

• Requires students to count orally starting with one
• Numbers that were not correctly counted in sequence are scored incorrect
• Typically a prompt at a three-second hesitation
Oral Counting

• **Screening**
  – Reliable
  – Concurrent and predictive validity

• **Monitoring**
  – Most sensitive to growth
  – Students most at risk in kindergarten and early first grade

Normative and criterion-referenced cut scores available from AIMSweb and mclassmath
Counting and Cardinality

- From saying counting words to counting out objects
  - Students usually know or can learn to say counting words prior to counting objects or telling the number of objects in a set
  - As students become fluent in saying the count sequence, they can focus attention on the pairings involved in counting objects

Progressions for the CCSS-M – Counting and Cardinality
Counting and Cardinality

- Usually initially facilitated by an indicating act (pointing to each object as the number word is said)
  - Learn to count objects (how many questions/quantity) presented in:
    - A line
    - Rectangular arrays
    - Circles
    - Scattered arrays
    - Counting out a given number of objects
Number Identification

• Requires a student to orally identify numbers (0-9, 0-20, 0-99)
• Numbers that are correctly identified counted as correct
• Typically a prompt at a three-second hesitation
Number Identification

• Screening
  – Most reliable
  – Concurrent and predictive validity

• Monitoring
  – Sensitive to growth
  – Students at risk in kindergarten and early first grade

Normative and criterion-referenced cut scores available from AIMSweb, mclassmath, DIBELSmath, ASPENS
Number Sense

“The discipline of mathematics comprises three worlds: the actual quantities that exist in space and time; the counting numbers in the spoken language; and formal symbols, such as written numerals and operation signs. Number sense requires the construction of a rich set of relationships among these worlds.”

Griffin, 2004
Sharon Griffin

Symbols

Counting Numbers

Quantity

\[ \begin{align*}
\text{one} & \quad \text{two} & \quad \text{three} \\
1 & \quad 2 & \quad 3 \\
\end{align*} \]

\[ X = \]

V. Faulkner and DPI Task Force adapted from Griffin

*NCSIP Foundations of Math slide*
Quantity Discrimination

- Requires students to name which of two visually presented numbers was larger
- Randomly sampled numbers (0-9, 0-20, 0-99)
- Typically a prompt at a three-second hesitation

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<td>16</td>
<td>43</td>
<td>57</td>
<td>4</td>
<td>12</td>
<td>37</td>
<td>48</td>
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Quantity Discrimination

- **Screening**
  - Reliable
  - Best concurrent and predictive validity
  - Floor effect in K

- **Monitoring**
  - Less sensitive to growth (2 months to make a decision)
  - Growth more closely associated with better outcomes

Normative and criterion-referenced cut scores available from AIMSweb, mclassmath, DIBELSmath, ASPENS
Counting and Cardinality

• Counting to Counting On
  – Students understand cardinality despite rearranging the set differently
  – Students can count on from a given counting word within a known sequence
  – Students understand that each successive number refers to a quantity that is one larger
Counting and Cardinality

- Once students begin to learn how the counting words relate to quantity, they begin to relate it to the written base-ten numerals
  - Difficulty for English speakers because of the incongruence with the system
Counting and Cardinality

- Comparisons of quantities, number names, numerals, and comparisons involving addition and subtraction (magnitude)
  - Match objects in two sets until seeing if one set has leftovers
  - Count the number of objects in each set and use their knowledge of the counting sequence to determine which set has more (which one goes farther along in the sequence)
  - Progresses to addition and subtraction questions involving “how many more” or “how many less”
Strategic Counting

- Requires students to name a missing number in a sequence
- Randomly sampled numbers (0-9, 0-20, 0-99)
- Typically a prompt at a three-second hesitation

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<th>12</th>
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<th>9</th>
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<th>11</th>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
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Strategic Counting

• **Screening**
  - Reliable
  - Good concurrent and predictive validity
  - Floor effect early K

• **Monitoring**
  - Less sensitive to growth (2 months to make a decision)
  - Possible non-linear growth

Normative and criterion-referenced cut scores available from AIMSweb, mclassmath, DIBELSmath, ASPENS
Gellman and Gallistel’s (1978) Counting Principles

- 1-1 Correspondence
- Stable Order
- Cardinality
- Abstraction
- Order-Irrelevance

*NCSIP Foundations of Math Slide*
Briars and Siegler (1984)
Unessential Features of Counting

- Standard Direction
- Adjacency
- Pointing
- Start at an End

*NCSIP Foundations of Math slide
**Students Identified with a Math Disability**

- **Pseudo-Error: Adjacency.**
  - Students with MD did not understand Order-Irrelevance

- **Error: Double Counts**
  - Students with MD/RD saw double counts as errors at end of sequence, but missed them at beginning of sequence.
  
  - **Working memory is a key factor!**

*NCSIP Foundations of Math slide
Diagnostic Assessment

• Number Knowledge Test
  – Developed by Sharon Griffin
  – Developmental Levels
  – Adequate reliability and validity
Preliminary

Let’s see if you can count from 1 to 10. Go ahead.

Number Knowledge Test Level 0 (4-year-old level)

1. Can you count these Counters and tell me how many there are?
   (Place 3 Counters in a row in front of the child.)

2a. (Show stacks of counters, 5 vs. 2, same color)
   Which pile has more?

2b. (Show stacks of counters, 3 vs. 7, same color)
   Which pile has more?

3a. This time, I’m going to ask you which pile has less.
   (Show stacks of counters, 2 vs. 6, same color.)
   Which pile has less?

3b. (Show stacks of Counters, 8 vs. 3, same color.)
   Which pile has less?

4. I’m going to show you some Counters.
   (Show a line of 3 blue and 4 white Counters in a row, as follows: BWBWBWW.)
   Count just the white Counters and tell me how many there are.

5. (Pick up all the Counters from the previous question. Show a mixed array—not a row—of 7 white and 8 blue Counters)
   Here are some more Counters.
   Count just the blue counters and tell me how many there are.
Number Knowledge Test Level 1 (6-year-old level)

1. If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?
2. What number comes right after 7?
3. What number comes two numbers after 7?
4a. Which is bigger: 5 or 4?
4b. Which is bigger: 7 or 9?
5a. This time, I’m going to ask you about smaller numbers. Which is smaller: 8 or 6?
5b. Which is smaller: 5 or 7?
6a. Which number is closer to 5: 6 or 2?
6b. Which number is closer to 7: 4 or 9?
7. How much is 2 + 4? (okay to use fingers)
8. How much is 8 take away 6? (okay to use fingers)
9a. When you are counting, which of these numbers do you say first? (show visual array 8526-ask child to point and say each numeral)
9b. When you are counting, which of these numbers do you say last?
Number Knowledge Test Level 2 (8-year-old level)

1. What number comes 5 numbers after 49?
2. What number comes 4 numbers before 60?
3a. Which is bigger: 69 or 71?
3b. Which is bigger: 32 or 28?
4a. This time I’m going to ask you about smaller numbers. Which is smaller: 27 or 32?
4b. Which is smaller: 51 or 39?
5a. Which number is closer to 21: 25 or 18?
   (Show visual array after asking the question)
5b. Which number is closer to 28: 31 or 24?
   (Show visual array after asking the question)
6. How many numbers are there in between 2 and 6?
   (Accept either 3 or 4)
7. How many numbers are there in between 7 and 9?
   (Accept 1 or 2)
8. (Show visual array 12 54.) How much is 12+54?
9. (Show visual array 47 21.) How much is 47-21?
   (use term take away)
Number Knowledge Test Level 3 (10-year-old level)

1. What number comes 10 numbers after 99?
2. What number comes 9 numbers after 99?
3a. Which difference is bigger, the difference between 9 and 6 or the difference between 8 and 3?
3b. Which difference is bigger, the difference between 6 and 2 or the difference between 8 and 5?
4a. Which difference is smaller, the difference between 99 and 92 or the difference between 25 and 11?
4b. Which difference is smaller, the difference between 48 and 36 or the difference between 84 and 73?
5. How much is 13 + 39?
   (Show visual array of 13 and 39.)
6. How much is 36 – 18?
   (Show visual array of 36 and 18.)
7. How much is 301 take away 7?

In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.
Diagnostic Assessment

- TEAM – McGraw Hill (Clements & Sarama)
- Mclass Beacon - Amplify
- I Ready – Curriculum Associates
Resources

- [http://maccss.ncdpi.wikispaces.net/](http://maccss.ncdpi.wikispaces.net/)
- *The Number Sense* – Dehaene
- *Understanding RTI in Mathematics* – Gersten & Newman-Gonchar
- *Assisting Students Struggling with Mathematics- Response to Interventions for Elementary and Middle Schools* - Gersten, R., Beckmann, Clarke, Foegen, Marsh, Star, & Witzel
- Learning and Teaching Early Math: A Learning Trajectories Approach- Clements & Sarama
- [www.turnonccmath.net](http://www.turnonccmath.net)
- [http://ime.math.arizona.edu/progressions/](http://ime.math.arizona.edu/progressions/)
- [www.mathinterventions.org](http://www.mathinterventions.org)
Application: Assessment of Number Sense and Adoption of a Research-Based Program

Rockwell Elementary School

Rowan County, NC
Rockwell Elementary School
Rowan County, NC

• RtI school beginning in 2009
• 26 classrooms to serve 525 students
• After analyzing universal screening data, we hypothesized on the Tier 1 plans that children were not proficient in basic computational skills due to lack of foundational number sense knowledge and instruction in core curriculum.
Number Worlds

• Rockwell researched number sense programs to assist with instruction.
• Number Worlds was chosen to be implemented in Kindergarten in 2012-2013 school year.
• Number Worlds instruction is 5 days a week for 30-45 minutes in ADDITION to the math block.
Cohort prior to number worlds Implementation 2011-2012 Kindergarten
Cohort prior to number worlds Implementation

2012-2013 1st grade

MCOMP 1st Grade

MCOMP 2nd Grade
1st Cohort of K who received number Worlds 2012-2013 data
1st Cohort who received Number worlds last year – Current 1st graders 2013-2014
How did we do it?

- Number Worlds trainers in the district came to Rockwell and completed a half day training.
- Kindergarten teachers also used the CD provided with the kit.
- Fidelity observations were and continue to be conducted three times a year to ensure fidelity of observations.